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Expand each of the expressions in Exercises 1 to 5.

1. $(1 - 2x)^5$

Solution:

From binomial theorem expansion we can write as

$(1 - 2x)^5$

$= {}^5C_o(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(2x)^5$

$= 1 - 5(2x) + 10(4x)^2 - 10(8x^3) + 5(16x^4) - (32x^5)$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

2. $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Solution:

From binomial theorem, given equation can be expanded as

$$\left(\frac{2}{x} - \frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^3 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$$

$$-{}^3C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^3C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^3C_5\left(\frac{x}{2}\right)^5$$

$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

3. $(2x - 3)^6$

Solution:

From binomial theorem, given equation can be expanded as

$$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_1(2x)^4(3)^2 - {}^4C_3(2x)^3(3)^3$$
$$= 64x^6 - 6\left(32x^5\right)(3) + 15\left(16x^4\right)(9) - 20\left(8x^3\right)(27)$$
$$+ 15\left(4x^2\right)(81) - 6(2x)(243) + 729$$
$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

4. $\left(\frac{x}{3} + \frac{1}{x}\right)^5$

Solution:

From binomial theorem, given equation can be expanded as

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^3C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^3C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2$$
$$= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5}$$
$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^3}$$

5. $\left(x + \frac{1}{x}\right)^6$

Solution:

From binomial theorem, given equation can be expanded as

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)'\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$
$$+ {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_3(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$
$$= x^4 + 6(x)^3\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

6. $(96)^3$

Solution:

Given $(96)^3$

96 can be expressed as the sum or difference of two numbers and then binomial theorem can be applied.

The given question can be written as 96 = 100 – 4

$(96)^3 = (100 – 4)^3$

$= {}^3C_0 (100)^3 – {}^3C_1 (100)^2 (4) – {}^3C_2 (100) (4)^2 – {}^3C_3 (4)^3$

$= (100)^3 – 3 (100)^2 (4) + 3 (100) (4)^2 – (4)^3$

= 1000000 – 120000 + 4800 – 64

= 884736

7. $(102)^5$

Solution:

Given $(102)^5$

102 can be expressed as the sum or difference of two numbers and then binomial theorem can be applied.

The given question can be written as 102 = 100 + 2

$(102)^5 = (100 + 2)^5$

$= {}^5C_0 (100)^5 + {}^5C_1 (100)^4 (2) + {}^5C_2 (100)^3 (2)^2 + {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100) (2)^4 + {}^5C_5 (2)^5$

$= (100)^5 + 5 (100)^4 (2) + 10 (100)^3 (2)^2 + 5 (100) (2)^3 + 5 (100) (2)^4 + (2)^5$

= 1000000000 + 1000000000 + 40000000 + 80000 + 8000 + 32

= 11040808032